A Practice Test

October 21, 2010

Name ____________________

1. (a) How many permutations are there of the letters $a, b, c, d, e, f, g, h, i$, and j?
   (b) How many start with the letter a?
   (c) How many of the permutations from (a) start with a vowel?
   (d) How many of the permutations from (a) have no adjacent vowels?
   (e) How many of the permutations from (a) have all vowels together?

2. (a) How many ways can 3 couples be arranged around a circular table. (Arrangements that differ by a rotation, are the same.)
   (b) In how many of these arrangements do the couples sit together.
   (c) In how many of the arrangements in (a) are exactly one couple seated together?

3. At the You-Mix-It trail mix store you get a choice between 5 kinds of nuts, 8 kinds of dried fruits, and 3 kinds of sweets. How many ways can you mix your You-Mix-It trail mix if
   (a) You must mix in 7 different items.
   (b) Of your 7 items, 2 must be nuts and 2 must be sweets.
   (c) Of the 7 items, exactly 1 is a sweet, and least 3 are nuts.

4. In a standard poker deck, (In each of 4 suits, there is one card of each of 13 ranks.) a poker hand consists of 5 unordered cards. How many ways can a player get a
   (a) Fullhouse? (Three cards of one rank and two of another.)
   (b) Two Pair and nothing higher? (Two cards of one rank, two of another, and one of a third rank.)

5. (a) How many integer solutions are there to the following

$$x_1 + x_2 + x_3 + x_4 = 8, x_i \geq 0, i = 1, \ldots, 4.$$

   (b) How many ways are there to distribute 20 indentical dimes among 4 children If the youngest must get at least one dime, the second youngest must get at least two dimes, the second oldest must get at least 3 dimes, and the oldest must get at least 4 dimes.

(c) For which positive integer $n \geq 19$ will the equations

$$x_1 + x_2 + \cdots + x_{19} = n, \text{ and}$$

$$x_1 + x_2 + \cdots + x_{64} = n,$$

have the same number of **positive integer solutions**. (Note that this question is a lot harder if you allow solutions with zero.)

6. Give all truth value assignments for the primitive statements p, q and r that make the following compound statement true

$$(p \to r) \to [(p \to q) \wedge (q \to r)].$$

7. Determine if $\neg(p \wedge \neg q) \to \neg p$ is a tautology.

8. (a) Two n-digit numbers (leading zeros allowed) are equivalent if one is a rearrangement of the other. (For example, 00123 and 10203 are equivalent.) How many 5 digit integers are not equivalent?
   (b) What if the digits $3, 5$, and 7 can occur at most once each?

9.
   (a) Using laws of inference, establish the validity of the argument $[(p \to (q \to r)) \wedge (\neg q \to \neg p) \wedge p)] \to r$
   (b) Show that the following argument is invalid.

$$\begin{array}{c} p \wedge \neg q \\ p \to (q \to r) \\ \hline \therefore \neg r \end{array}$$

10. Show that

$$\neg \forall x \forall y [p(x, y) \to q(x)] \iff \exists x \exists y [p(x, y) \wedge \neg q(x)].$$

11. Let $A = \{1, 2, \ldots, 20\}$. **This isn't in the scope of the test.**
   (a) What is $|\{x \mid (x \in A) \wedge (x \text{ is prime})\}|$?
   (b) Give the set $\{x \mid (x \in A) \wedge (5x \in A)\}$.
   (c) What is $|\mathscr{P}(A)|$?
   (d) What is $|\mathscr{P}(\mathscr{P}(A))|$?
   (e) What is the cardinality of $\{B \mid (B \in \mathscr{P}(A)) \wedge (|B| = 4)\}$?