Discrete Math Test 1

Each question is worth 10 points, distrubited evenly among the parts.

1. (a) How many arrangements (permutations) are there of the letters of the word "REDRESSES"?
   (b) How many arrangements are there in which all of the E's are together?
   (c) How many arrangements contain the word "DRESS" in that order?
   (d) How many of the arrangements in (a) have no consecutive E's?

2. (a) A fruit stand has apples, blueberries, coconuts, dates, and elderberries. How many ways can you choose a selection of 20 different fruits?
   (b) How many ways can you do this if you must have at least one of each kind?
   (c) How many integer solutions are there to $x_1 + x_2 + x_3 + x_4 \leq 40$ when $x_i \geq 1$ for $i = 1, \ldots, 4$? (Notice that it is "≤ 40", not "$= 40$".)

3. (a) On the euclidean plane how many distinct paths are there from $(0, 1)$ to $(5, 5)$ using only the following movements
   - $(x, y) \to (x + 1, y)$
   - $(x, y) \to (x, y + 1)$

   (b) How many of these paths don't use the move $(2, 3) \to (3, 3)$.

4. (a) Construct a truth table for the statement $[p \wedge (p \to q)]$.
   Is the statement a tautology?
   (b) Verify that $[p \vee (q \wedge r)] \vee \neg[p \vee (q \wedge r)]$ is a tautology. (This should be really quick.)

5. (a) Using laws of inference, establish the validity of the argument $[(p \to q) \wedge (r \to \neg q) \wedge r)] \to \neg p$
   (b) Show that the following argument is invalid.

$$\begin{array}{r} p \leftrightarrow q \\ q \to p \\ r \vee \neg s \\ \neg s \to q \\ \hline \therefore s \end{array}$$

6. Given the statements

   $p(x)$: $(x - 2)(x - 5) = 0$
   $r(x)$: $x < 0$

   Determine the truth or validity of the following statements, where the universe for x and y is all integers. (Give explanations.)

   (a) $\forall x[p(x) \to \neg r(x)]$
   (b) $\exists x[p(x) \to r(x)]$
   (c) $\forall x \exists y[p(x) \to r(y)]$
   (d) $\exists x \forall y[p(x) \wedge (x + y = 1)]$