Linear Algebra 2 2019 Final Test

- Hand your solutions in by Dec 13 (Friday).
- Write neatly, especially your name and student number.
- You are allowed to use computers to compute and matrix products, inverses, eigenvalues, or diagonalisations. If you use a computer, be clear about what you have done. I don't want just the anwers. (You can use www.wolframalpha.com or www.cocalc.com to compute these things.)
- There are 5 questions worth 5 points each for a total of 25.

1. (a) Show that if A (invertible, but not necessarily symmetric) has eigenvalues 3 and 1 then the condition number of A is at least 3.

   (b) What can you say about the condition number of A^{-1}?

2. Find a tridiagonal matrix that is similar to $A = \begin{bmatrix} 1 & 3 & 4 & 5 \\ 3 & 3 & 1 & 0 \\ 4 & 1 & 3 & 3 \\ 5 & 0 & 3 & 0 \end{bmatrix}$.

3. Do one iteration of the QR-method for finding the eigenvalues of $A = \left[\begin{smallmatrix} 3 & 1 \\ 1 & 4 \end{smallmatrix}\right]$. (Use any method you want for the QR-decomposition.) What are your approximations of the eigenvalues of A?

4. Do two iterations of the Gauss-Seidel method, with initial guess $x_0 = (1, 1)$, to approximate the solution to $\left[\begin{smallmatrix} 2 & 3 \\ 1 & 2 \end{smallmatrix}\right]x = \left[\begin{smallmatrix} 3 \\ 2 \end{smallmatrix}\right]$.

5. For the following problem, use the simplex method to a) solve an auxillary problem to find the first feasible corner, and then b) solve the problem.

   Minimize the cost function $x_1 + 2x_2$ over all positive solutions of

$$\begin{aligned} x_1 + 4x_2 &\geq 8 \\ 2x_1 + x_2 &\leq 8 \end{aligned}$$

Question	Out of	Score
1	5	
2	5	
3	5	
4	5	
5	5	
Total	25	