Linear Algebra 2019 Final Test

Name ____________ Student # ____________

- Write neatly, especially your name and student number.
- Clearly $\boxed{\text{box}}$ your final answer if there is one.
- There are 12 questions worth 5 points each for a total of 60.

1. Answer the following with True (T) or False (F).

   (a) If $x \in W$ then $\operatorname{proj}_W x = 0$.

   (b) If $S = T^\perp$ then $T = S^\perp$.

   (c) The projection matrix for the space generated by $(1, 0, 0)$ and $(1, 1, 0)$, is invertible.

   (d) If $\det(M) = 4$ then $\det(M - I) = 3$.

   (e) The cofactor matrix of an invertible matrix M has determinant 1.

	T or F
a	
b	
c	
d	
e	

2. Find a basis for the orthogonal complement $A^\perp$ of the space A spanned by $(1, 2, 2, 3)$ and $(1, 3, 3, 2)$.

3. Find the matrix that projects every point in the plane $\mathbb{R}^2$ onto the line $x - 3y = 0$. What is the projection p of the point $(1, 2)$ onto this line?

4. Find a least squares approximation $\hat{x}$ to a solution of $Ax = b$ where

$$A = \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 2 & 3 \end{bmatrix} \qquad b = \begin{bmatrix} -5 \\ 8 \\ 1 \end{bmatrix}.$$

5. If P is the projection onto the column-space of A, what is the projection onto the left nullspace? (Why?)

6. Let M be an $n \times n$ matrix whose columns are pairwise orthonormal. If every entry of M is $1/4$ or $-1/4$, then what is n? (Explain why.)

7. Let $a = (2, 2, 2)$, $b = (1, -1, 0)$ and $c = (2, 0, 1)$.

   (a) Show that a and b are orthogonal.

   (b) Show that c is in the space generated by a and b.

   (c) Find an orthogonal basis, containing a and b, for the space generated by a, b, and $d = (1, 2, 3)$.

8. If a 4×4 matrix M has $\det(M) = 1/3$ what are

   (a) $\det(2M)$

   (b) $\det(M^{-1})$

   (c) $\det(-M)$

   (d) $\det(M^T M)$

	Answer
a	
b	
c	
d	

9. (a) What does it mean for a matrix Q to be orthogonal?
   (b) Show that an orthogonal matrix Q has determinant -1 or 1.
   (c) Describe the parallelopiped whose edges are the column vectors of an orthogonal matrix Q.

10. Find the determinants of

   (a) $\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 1 & 2 & 2 \\ 4 & 4 & 8 & 8 \end{bmatrix}$

	Answer
a	
b	
c	

11. Use Cramer's rule to find x_2 where $\begin{bmatrix} 2 & 5 & 7 \\ 0 & 1 & 4 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$.

12. Find the area of the triangle with vertices (corners) at the points $(2, 3)$, $(4, 1)$ and $(6, 4)$.

Question	Out of	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
11	5	
12	5	
Total	60	