Linear Algebra 2019 Midterm Test

Name _____ Student # _____

- Write neatly, especially your name and student number.
- Clearly $\boxed{\text{box}}$ your final answer if there is one.
- There are 8 questions. Each question is worth 5 points, for a total of 40.

1. Answer 'T(rue)' or 'F(alse)'. [/8]

   (a) If $M^2 + M = I$ then $M^{-1} = M + I$.

   (b) If the diagonal entries of a matrix M are 0, then M is singular.

   (c) A matrix M is invertible if and only if it has 3 pivots.

   (d) The matrix $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ has rank 2.

   (e) There are n different $n \times n$ permutation matrices.

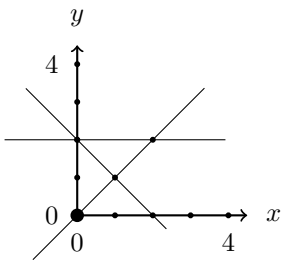
   (f) The LU decomposition of a matrix is unique if it exists.

   (g) Any three independent vectors in $\mathbb{R}^3$ span $\mathbb{R}^3$.

   (h) Given a matrix A, if we reduce the augmented matrix $[A \mid I]$ by Gaussian Elimination to $[A' \mid B]$, then $BA = A'$.

	T or F	Comment/Reason
a		
b		
c		
d		
e		
f		
g		
h		

2. Given a system of three equations in two variables (x and y), where the equations define the shown lines, find all solutions to the system. [/3]



3. Find a, b and c where [/3]

$$\begin{bmatrix} a & 2 \\ 1 & 1 \\ b & 4 \end{bmatrix} \begin{bmatrix} c \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix}.$$

4. (a) Write out the 3×3 row operation matrix E_1 that adds three copies of row 1 to row 2.

   (b) Write out the 3×3 row operation matrix E_2 that switches row 1 with row 3.

   (c) Compute E_1^n, and E_2^n.

[/3]

5. Find an LDU decomposition of the matrix [/3]

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 4 & 7 & 2 \end{bmatrix}.$$

6. Find the inverse of the following matrix. [/3]

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 \\ -2 & 1 & 1 & -3 \\ -1 & 3 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

7. Which of the following subsets of $\mathbb{R}^3$ are subspaces: [/6]

(a) The set of vectors (b_1, b_2, b_3) with $b_1 = 1$.

(b) The set of vectors (b_1, b_2, b_3) with $b_1 = 0$.

(c) The set of vectors (b_1, b_2, b_3) where b_1 is integer.

(d) The set of vectors (b_1, b_2, b_3) with $b_1 b_2 = 0$.

(e) The set of linear combinations of the vectors $(1, 2, 6)$ and $(3, 2, 1)$.

(f) The set of vectors (b_1, b_2, b_3) with $b_1 + b_2 - 2b_3 = 3$.

8. Construct a 3×3 matrix whose column space contains $(1, 1, 0)$ and $(1, 0, 1)$ but not $(1, 1, 1)$. [/3]

9. Where A is the matrix in question 6, solve $A[w, x, y, z]^T = [2, 3, 4, 5]^T$. [/3]

10. The matrix equation $M\mathbf{x} = \mathbf{b}$ has solutions $(1, -1, 2)$ and $(-2, 2, -4)$. What is $\mathbf{b}$? [/3]

11. How would you decide if a set of vectors of length 3 are independent? Do it for the following vectors: [/3]

$$(1,3,2), (2,1,3), \text{ and } (3,2,1).$$

12. Where [/5]

$$C = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

(a) Find the solution of $Cx = 0$

(b) Find the solution of $Cx = [1,2,3]^T$.

(c) Give a basis of the columnspace of C.

(d) Give a basis of the rowspace of C.

(e) What is the rank of C?

13. (a) What matrix transforms $(1, 0, 0)$ to $(2, 3, 4)$, $(0, 1, 0)$ to $(1, -1, 2)$ and $(0, 0, 1)$ to $(1, 1, 1)$?

    (b) What does this same matrix transform $(1, -1, 2)$ to?

[/3]

14. Which of these transformation **are** linear? Where $v = (v_1, v_2)$, [/3]

    (a) $T(v) = (v_2, v_2)$.

    (b) $T(v) = (1, v_2)$.

    (c) $T(v) = (-v_2, v_1 + v_2)$.

    (d) $T(v) = (v_1^2, -(v_2^2))$.

15. Let A and B be the matrices such that [/3]

$$A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ and } B\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

What are the products AB and BA.

Question	Out of	Score
1	8	
2	3	
3	3	
4	3	
5	3	
6	3	
7	6	
8	3	
9	3	
10	3	
11	3	
12	5	
13	3	
14	3	
15	3	
Total	55	