

Applied Algebra 2020 (Take Home)

Test 1

- Write neatly!
- For full marks, show work and give reasons.
- There are 4 questions, each worth 10 points. (The first two are short and easy, the third takes an idea, and writing it up should be about half a page. The last is not really hard, and we saw how to do it in class, but writing part (c) may be messy and should take at least half a page. The grading is on how clear you can make it. Choose your steps and your names carefully to make it nice.)
- Do not discuss problems with other students! Points will be docked for SUSPECTED copying.
- Return completed test to me by 5pm on Tuesday September 29, in an email with subject line ‘Math 426 Test 1’.
- Make sure your name and student number are in the body of the email.
- Name your pdf StudentNumber-Name.pdf. If you send several picture files name them sequentially: StudentNumber-Name-01.jpg, StudentNumber-Name-02.jpg, etc.

1. Show that for any structure \mathcal{H} , all polymorphisms of the structure \mathcal{H}^u are idempotent.
2. Give an example of a non-bipartite non-core graph. (Choose one that is unlikely to be the same as anyone else’s. You lose a mark for every person that has the same one.)
3. Where \vec{P}_n is the directed path on the set $\{0, \dots, n\}$ with arcs $\{(i, i+1) \mid i \in \{0, \dots, n-1\}\}$ and \vec{T}_n is the transitive tournament on the set $\{0, \dots, n\}$ in which (i, j) is an arc if and only if $i < j$, show that for any digraph H

$$\vec{P}_{n+1} \not\rightarrow H \iff H \rightarrow \vec{T}_n.$$

Explain how this gives a polynomial time algorithm for $\text{CSP}(\vec{T}_n)$.

4. Recall that K_3 , as a relational structure has vertex set $\{0, 1, 2\}$ and one symmetric binary relation E . The structure K_3^s has three more unary relations.

- (a) Explicitly give all the relations of K_3^c .
- (b) Show that the unary relation $S = \{0, 1\}$ can be *pp*-defined from the relations of K_3^c .
- (c) Show that the 3-ary relation

$$R_{001} = \{0, 1\}^3 \setminus \{(0, 0, 1)\}$$

is *pp*-definable from the relations of K_3^c . (Hint: Look at HW Problem 2.1.4. Break it up into a couple of steps. When you use a *pp*-formula to define a relation, tell me what the relation is, but don't worry about proving it.)