2022 Probability and Statistics Practice Test

Name _____ Student # _____

Each question is worth 5 points. Don't evaluate anything that is unreasonable!

1. A bowl contains nine blue chips and one red one. You draw chips, without replacement, until you draw the red one. X is the number of chips you draw. What is the pmf p of X? What if you replace the chip removed after each draw?

2. Let C_1, C_2, C_3 be mutually independent events. Show that $C_1 \cup C_2$ and C_3 are independent. (You may use the fact that C_i, C_j^c and C_k are also mutually independent for distinct i, j and j,)

3. Let C_1, C_2, C_3 be mutually independent events with probabilities $1/2, 1/5$ and $1/5$ respectively. What is $P(C_1 \cup C_2 \cup C_3)$?

4. Let X have the pdf $f(x) = (x+2)/c$ for some constant c and support $-2 < x < 3$.

   (a) Find c.

   (b) Find $P(X^2 < 4)$.

5. Find the mean and variance of the distribution with cdf

$$F = \begin{cases} x/8 & \text{if } 0 \le x < 2 \\ x^2/16 & \text{if } 2 \le x < 4 \end{cases}$$

   on the support $[0, 4]$.

6. Let X have pdf $f(x) = 6x(1-x)$ for its support $(0, 1)$. Let $\mu = E(X)$ and $\sigma^2 = \text{Var}(V)$.

   (a) Bound $P(\mu - 2\sigma < X < \mu + 2\sigma)$ using Chebyshev's inequality.

   (b) Find $P(\mu - 2\sigma < X < \mu + 2\sigma)$ exactly.